Parrondo's Games

Home | Up

Parrondo's Games
Stochastic Resonance
Publications

Brownian Ratchets, Parrondo's Games and more...

Two games with unusual properties were originally devised by Prof. Juan MR Parrondo as a pedagogical example of a Brownian ratchet. A few short descriptions about different topics are described below.

Feynman's ratchet and pawl as a basis for the Brownian ratchet

A ratchet and pawl device, shown in Figure 1, was introduced in the last century as a proposed perpetual motion machine -- the aim was to try and harness the thermal Brownian fluctuations of gas molecules, by a process of rectification.

The device is considered to be of molecular scale, and works in the following manner. Let the temperature of the thermal bath in the boxes be equal so T1 = T2 = T. Hence, the energy, which is directly related to the temperature of the thermal baths, is also equal in each bath. Due to the bombardments of gas molecules on the vane, it oscillates and jiggles. Since the wheel at the other end of the axle only turns one way, motion in one direction will cause the axle to turn while motion in the other direction will not. Thus the wheel will turn slowly and may even be able to lift some weight.

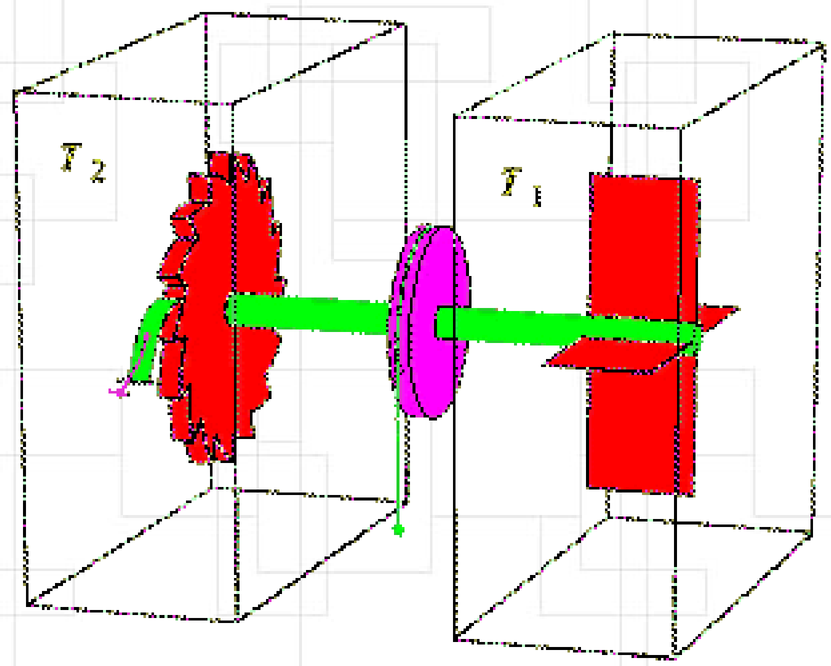


Figure 1. The ratchet and pawl machine. There are two boxes with a vane in one and a wheel that can only turn one way, a ratchet and pawl, in the other. Each box is in a thermal bath of gas molecules at equilibrium. The two boxes are connected mechanically by a thermally insulated axle. The whole device is considered to be reduced to microscopic size so gas molecules can randomly bombard the vane, to produce motion.

This is a violation of the Second Law of Thermodynamics.

This creates a paradox, the ratchet and pawl will apparently work in perpetual motion when T1 = T2. However, at equilibrium the effect of thermal noise is symmetric, even in an anisotropic medium. The Second Law implies that structural forces alone cannot bias Brownian motion as has been suggested with the ratchet and pawl device.

How is this possible? What is the explanation?

The focus of recent research is to harness Brownian motion and convert it to directed motion, or more generally, a Brownian motor, without the use of macroscopic forces or gradients. This research was inspired by considering molecules in chemical reactions, termed molecular motors.

The roots of these Brownian devices trace back to Feynman's exposition of the ratchet and pawl system. By supplying energy from external fluctuations or non-equilibrium chemical reactions in the form of a thermal or chemical gradients, directed motion is possible even in an isothermal system. These types of devices have been shown to work theoretically, even against a small macroscopic gradient. Recently, with the technology available to build micrometer scale structures, many man-made Brownian ratchet devices have been constructed, and actually work.

Page last updated 13 July, 2000

Page by GPH

The mechanism of the flashing Brownian ratchet

Using the ratchet and pawl machine as a basis for a Brownian ratchet, several ideas have been used to get directed movement from Brownian particles. A common mechanism, referred to as a flashing ratchet, which may prove fruitful when comparing it to Parrondo's games, is described below.

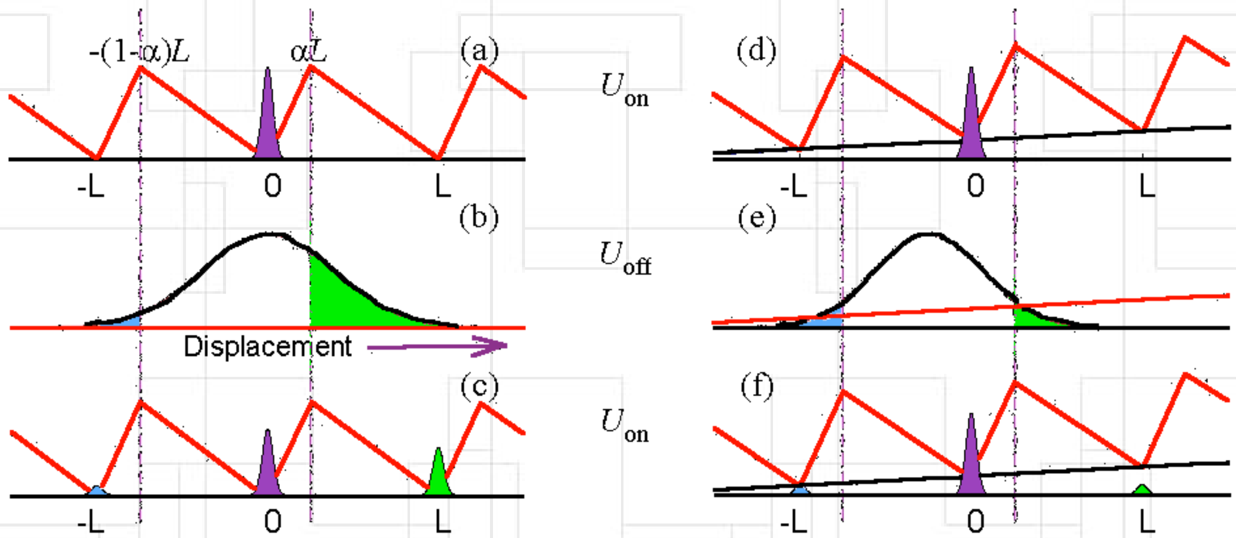


Figure 2. This shows the how the mechanism of the ratchet potential works. The diagrams on the left, (a)-(c) shows when there is no macroscopic gradient present and the net movement of particles is in the forward direction (defined by arrow). The diagrams on the right, (d)-(f) have a slight gradient present, this causes the particles to drift backwards while Uoff is acting. Hence the net flow of particles in the forward direction is reduced.

Consider a system where there exists two one-dimensional potentials, Uon and Uoff, as shown in Figure 2. The asymmetry of the potential Uon is determined by a, where 0 < alpha < 1. Having alpha = 1/2 creates a triangular symmetric potential otherwise the potential is asymmetrical like Uon in Figure 2 where alpha < 1/2. Let there be Brownian particles existing in the potential diffusing to a position of least energy.

When the Uon is applied, the particles are trapped in the minima of the potential so the concentration of the particles is peaked. Switching the potential off allows the particles to diffuse freely so the concentration is a set of Gaussian curves centered around the minima. When Uon is switched on again there is a probability Pfwd that is proportional to the green area of the curve that some particles are to the right of alpha*L. These particles move forwards to the minima located at L. Similarly there is a probability Pbck for particles that are to the left of (1-alpha)L. Since a < 1/2 then Pfwd>Pbck the net motion of the particles is to the right. Thus we have directed motion!!

Here is a good Java applet that shows a flashing BR working and allows you to change the parameters.

Ratchet and Pawl
Brownian Ratchet
Parrondo's Games
Mathematical Analysis
Parameter Space
Entropy

Next Page

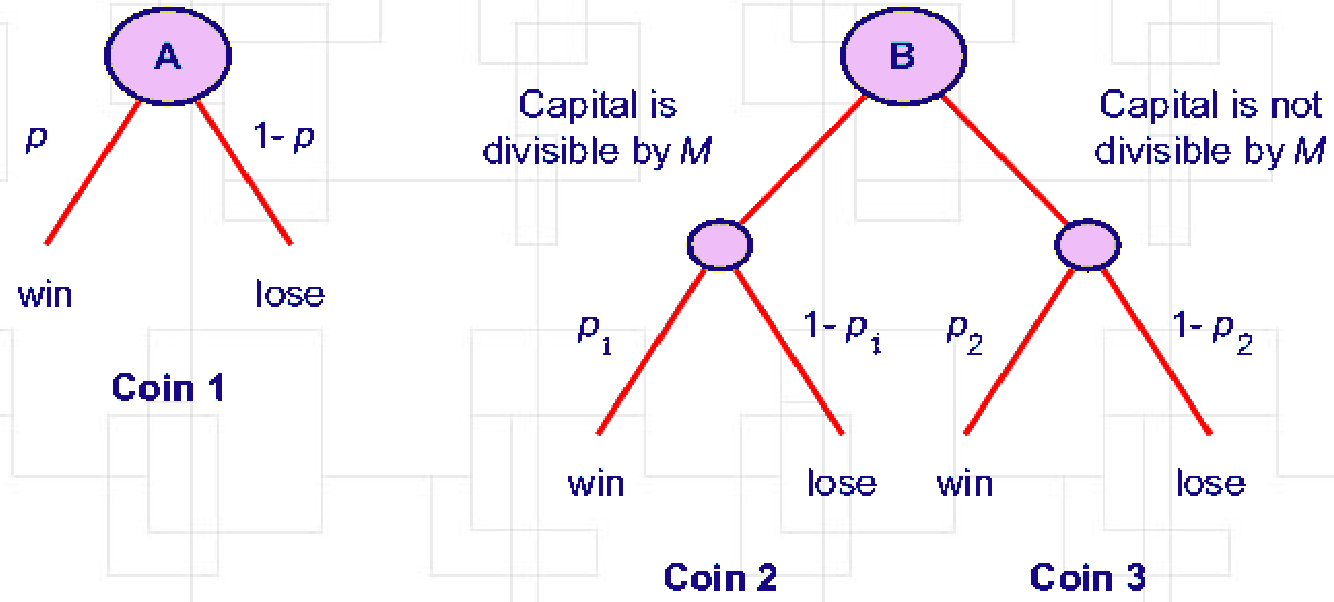
Previous Page

Relevant Papers
Press Articles

Index

Parrondo's Games

Here we introduce two games, A and B. Game A can be described like tossing a coin, or going on a biased random walk, (with probability p). Game B is a little more complex and can be generally described by the following statement. If the present capital is a multiple of M then the chance of winning is p1, if not, the the chance of winning is p2. The games are shown graphically below in terms of tossing biased coins.



A convenient parameterisation is given in terms of a biasing parameter epsilon via the transformation p = p' - epsilon, p1 = p'1 - epsilon and p2 = p'2 - epsilon. Using Parrondo's original parameter values, we have p = 1/2 - epsilon, p1 = 1/10 - epsilon, p2 = 3/4 - epsilon, M = 3 and epsilon = 0.005.

If we run some simulations playing these games, we find that when played individually the two games lose when epsilon > 0. But Consider the scenario if we start switching between the two losing games, play two games of A, two games of B, two of A, and so on. The result, which is quite counter intuitive, is that we start winning. That is, we can play the two losing games A and B in such a way as to make a winning outcome. The result of playing the games is shown in Figure 3.

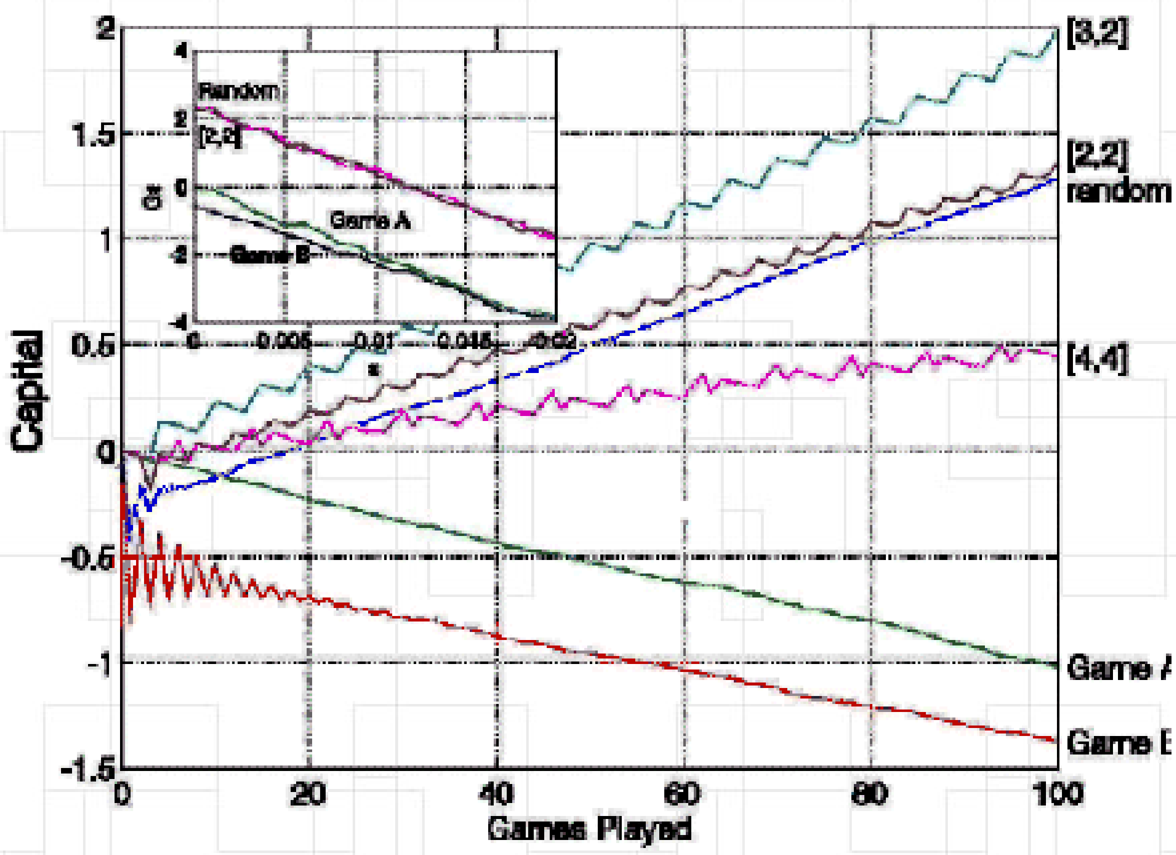


Figure 3. The main plot shows the effect of switching between games A and B. The simulation was performed by playing game A a times, then game B b times, game A a times, and so on with epsilon = 0.005 and averaged over 50 000 runs. The inset shows the effect of the games performance when varying epsilon -- the outcome after the 100th game is plotted.

Note on Figure 3. The notation v =[a,b] was introduced to indicate that game A is played a times, then game B played b times, and so on.

Now comparing the results of these games with that of how the Brownian ratchet works, and using our knowledge of the simple, well known game A, we may be able to deduce what is going on here. Well, I wouldn't want to spoil your fun, so I'll let you work it out.

If we are getting directed movement of the capital, where is the energy coming from?

Ratchet and Pawl
Brownian Ratchet
Parrondo's Games
Mathematical Analysis
Parameter Space
Entropy

Next Page

Previous Page

Relevant Papers
Press Articles

Index

A mathematical analysis of the games using discrete-time Markov chains.

So how can two losing games, when combined randomly produce a winning game. We can show a mathematically analysis that establishes this paradox. By establishing the conditions for recurrence of the corresponding discrete-time Markov chain (DTMC) it can be show that probabilities can be chosen such that games A and B lose when played individually, but win when played randomly. (The DTMC analysis was provided by Peter Taylor, Dept. of Appl. Maths.)

Here, just a summary of the analysis is presented, a more detailed description is available in the relevant papers.

The player wins a single round of game A with probability p and loses with probability 1-p. The analysis for game A is quite elementary, and the result which accords with our intuition is that we lose if

$$\frac{1-p}{p} > 1. \qquad (1)$$

Now let us turn to game B. Here the probability that the player wins a single round depends on the value of their current capital. If the capital is a multiple of M, the probability of winning is p1, whereas if the current capital is not a multiple of M, the probability of winning is p2. The corresponding losing probabilities are 1-p1 and 1-p2 respectively. The analysis for game B is that we lose if

$$\frac{(1-p_1)(1-p_2)^{M-1}}{p_1 p_2^{M-1}} > 1. \qquad (2)$$

Now consider the situation where the player plays game A with probability g and game B with probability 1 - g (g for gamma). If our capital is a multiple of M the probability of that we win the randomised game is q1=gp + (1-g) p1, whereas if our capital is not a multiple of M the probability that we win is q2=gp + (1-g) p2. The probabilities of losing are 1 - q1 and 1 - q2 respectively. We observe that this is identical to game B except that the probabilities have changed. It follows from (2) that the randomised game is winning if

$$\frac{(1-q_1)(1-q_2)^{M-1}}{q_1 q_2^{M-1}} < 1. \qquad (3)$$

Thus, the existence of the paradox of Parrondo's games will be established if we can find parameters p, p1, p2, g and M for which the three above equations are satisfied. For example p = 5/11, p1 = 1/121, p2 = 10/11, g = 1/2 and M = 3 then we have 6/5 > 1, 6/5 > 1 and 217/300 < 1 for the three equations respectively. With these parameters, games A and B are losing, but the randomised game is winning.

A similar type of analysis can be employed that gives the rate of change of capital with respect to games played, i.e. the slope if the lines in Figure 3. This also determines if the game(s) are winning or losing depending on whether the rate of change is positive or negative respectively. In addition we can determine the expected capital to be achieved after a certain number of games have been played.

Game B is losing -- A common fault

If we consider game B prima facie, we may intuitively guess from a statistical point of view that coin 2 is played, on average, 1/3 of the time, and coin 3 the remaining 2/3 of the time. (Since we are working on capital modulo three). Under these assumptions, the probability of winning game B is 1/3*(0.1-0.005) + 2/3*(0.75-0.005) = 0.5283, which is greater than 0.5, thus the game is winning. Wrong.

In this scenario we have only considered the games from a statistical point of view, and not correctly as discrete-time Markov chains. Hence, our guess for the equilibrium distribution of y = x mod 3 in game B is not correct.

It is easy to see that y is the state of a three state Markov chain with transition probabilities given by p(1,2) = p(2,0) = 3/4, p(1,0) = p(2,1) = 1/4, p(0,2) = 9/10 and p(0,1) = 1/10. See Figure 3b.

Using discrete-time Markov chain (DTMC) theory, the equilibrium distribution of this chain is (pi_0, pi_1, pi_2) = (0.3846, 0.1538, 0.4615) when epsilon = 0. The calculation for winning game B then reduces to 0.5, which means that game B is fair.

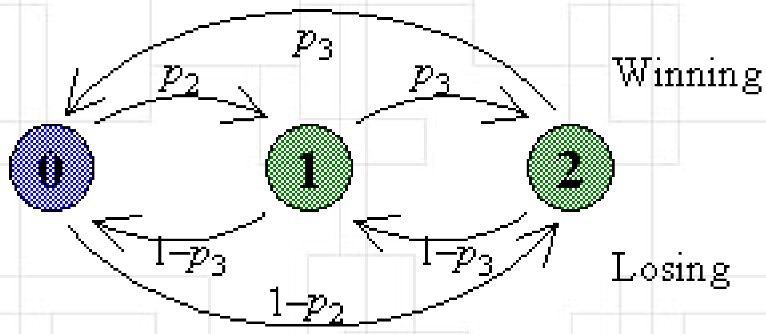


Figure 3b. Three state Markov chain, formed by y = capital mod 3.

When a bias of epsilon = 0.005 is included the equilibrium distribution changes slightly to (pi_0, pi_1, pi_2) = (0.3836, 0.1543, 0.4621). The probability of winning game B then reduces to [0.3836]*(0.095) + [0.1543 + 0.4621]*(0.745) = 0.4957. So, in fact, game B is definitely losing when epsilon = 0.005. If you are unfamiliar with Markov chain analysis, you can satisfy yourself of these results via computer simulation of game B.

Ratchet and Pawl
Brownian Ratchet
Parrondo's Games
Mathematical Analysis
Parameter Space
Entropy

Previous Page
Next Page

Relevant Papers
Press Articles

Index

Parameter Space for Parrondo's games.

Game A is simple and only depends on p as dictated by (1). This simplifies to $p < 1/2$.

The condition for winning or losing game B depends on parameters p1 and p2 (for a given value of M). This can be represented in two-dimensional space as shown in Figure 4 for various values of M. We deduce that the area above the lines creates winning games while the area below the lines creates losing games. The points shown by the triangles and diamonds where found by trial and error from numerical simulations. These values agree well with the analytical solution developed.

Now consider the randomised game, the winning or losing conditions are dependent on three parameters p, p1 and p2 (for given gamma and M). This can be represented in three-dimensional (3-D) space using axes {p, p1, p2}, as can games A and B.

Well, I don't want to spoil your fun any more. If you draw the surfaces for each of the games you will find that there is an enclosed region that defines the parameter space for Parrondo's paradox. That is, choose any point in that enclosed region and Parrondo's paradox will exist.

OK, I lied. Figure 5 shows the surfaces for each of the games. the surface denotes a boundary between winning and losing regions.

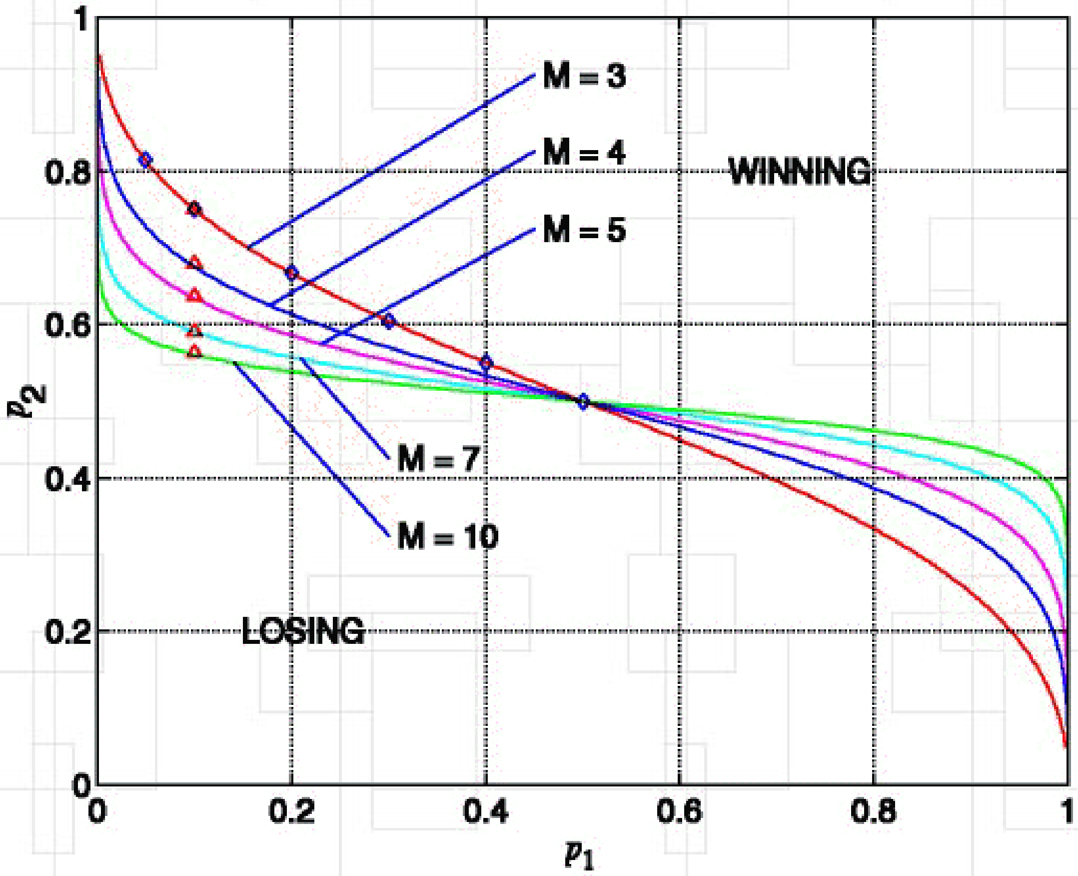


Figure 4. The solid lines show the values of p1 and p2 satisfying (2) to make game B fair. A continuous range of possibilities exist for each value M.

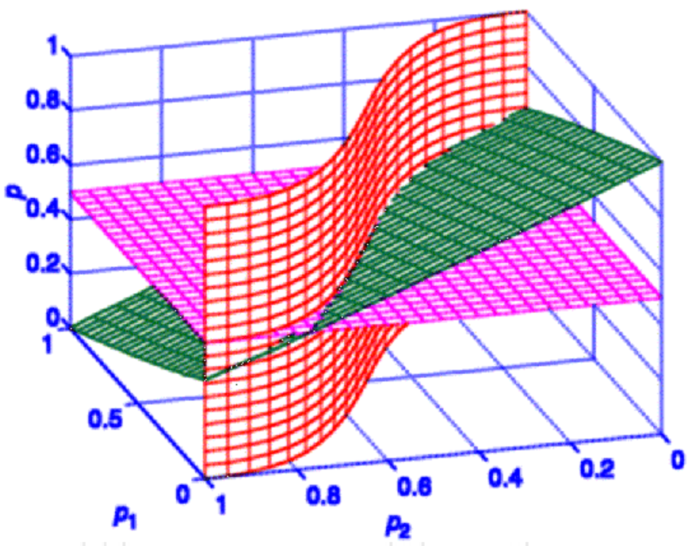


Figure 5. Parameter space for Parrondo's games when M = 3. The surfaces represent the boundary between winning and losing games. the magenta plane is game A, red surface is game B, and dark green surface is the randomised game. the other constraints are obviously the boundary conditions, the probabilities must be in [0,1]. The small volume in the foreground shows the parameters where Parrondo's paradox exists. There is also a corresponding volume in the opposite corner, where the opposite to Parrondo's paradox exists.

Next Page

Previous Page

Entropies of Parrondo's games.

Now let us think of the sequences of wins and losses of the games in terms of information theory. Denote by Xj the random variable which represents the outcome at time point j when playing any of the games A, B or C. If the game wins at the j th time point, then Xj = 1, if it loses then Xj = 0. This results in a binary chain.

1	1	0	1	0	0	1	0	1	...

For the case of game A, the chain is totally uncorrelated, thus the entropy of the chain can be found from standard information theory as

$$H(p) = -p\log_2 p - (1-p)\log_2(1-p) \cdot \qquad (4)$$

This gives the stock standard entropy plot found in many text books, with a maximum of 1 bit at p = 1/2.

The random variables Xj for game B is correlated and so the expressions for the entropy rate is more complicated. To find the entropy rate of game B, we need to calculate the equilibrium probabilities pi_i^B that the capital is congruent to i modulo M for i = {1, ... , M}. After some algebra, the entropy, taking into account correlations, is given by

$$\frac{1-p}{p} > 1 \cdot \qquad (5)$$

We can also calculate the entropy rate as if the random variable Xj was independent.

$$\frac{(1-p_1)(1-p_2)^{M-1}}{p_1 p_2^{M-1}} > 1 \cdot \qquad (6)$$

Calculating the entropies for game C uses exactly the same theory as for game B, except all the p's are replaced by q's as given in the math analysis page. The equilibrium distributions obviously needs to be recalculated again as well (using appropriate q's).

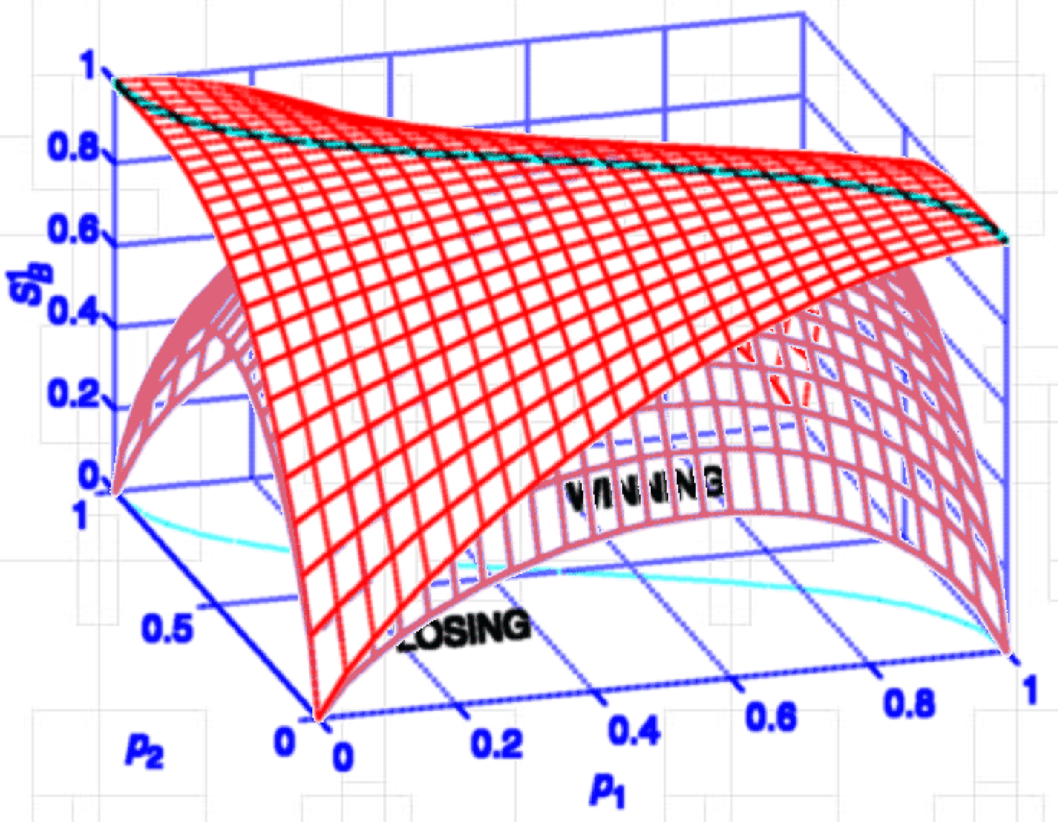


Figure 6. The entropies of game B. The purple surface shows the entropies taking into account correlations, that is, generated from (5). The red surface are the entropies not taking into account any correlations in the chain. As we expect, entropies taking into account the correlation, that is, knowledge of the source, are always less than or equal to those not taking into account any correlations that may exist.

The argument of H on the right hand side of (6) is simply the stationary probability of producing a 1 at a given time point of game B. That is, it is simply the entropy taking the chain to be uncorrelated.

Since H(p) is concave, it is easy to see that S^1_B >= S^B and S^1_C >= H^C, the former is shown in Figure 6. Although the later is a different shape, it contains the same characteristics.

If we consider the entropy at (p1,p2) = (0.9,0.1) for example, the chain will contain approx the same number of ones and zeros. So from the definition of entropy H(p) in (4) we might expect to entropy to close to 1 bit. But it more like about 0.2 bits. The answer is that we haven't considered the fact that we know how the chain was generated. Given (0.9, 0.1) and the result of the last game, we can predict with reasonably high accurately what the next result is going to be, thus uncertainty is reduced, i.e. the entropy.

Clearly, the entropies of the games are related to the parameter space of the games, but we won't go into that here.

What we have in Figure 6 are the two extreme entropies. The top surface only considers one bit in the chain at a time, while the bottom surface considers every bit in the chain. What if we consider something in between?

We may denote lambda to indicate the number of bits (in the chain) that we are considering, i.e. the size of the window that we are passing over the chain. It is known that the semi-uncorrelated entropies converge to the correlated entropy as lambda approaches infinity. This is shown from simulations in Figure 7, the algebraic expressions get messy for anything other than (5) and (6) above.

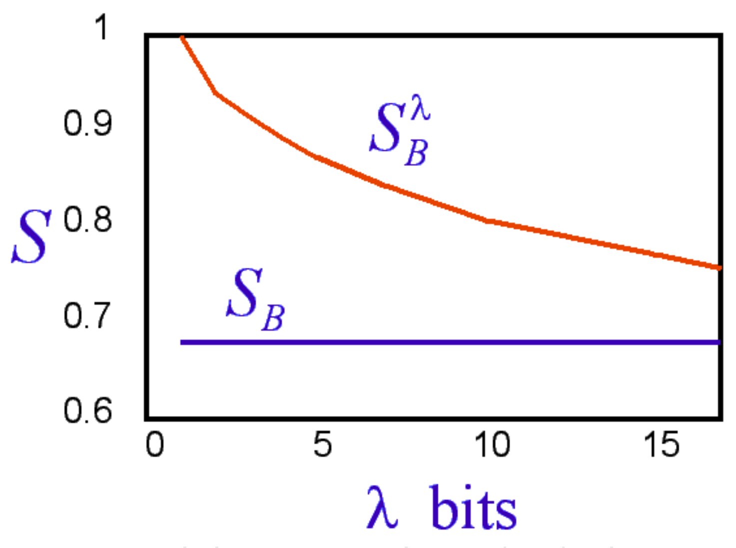
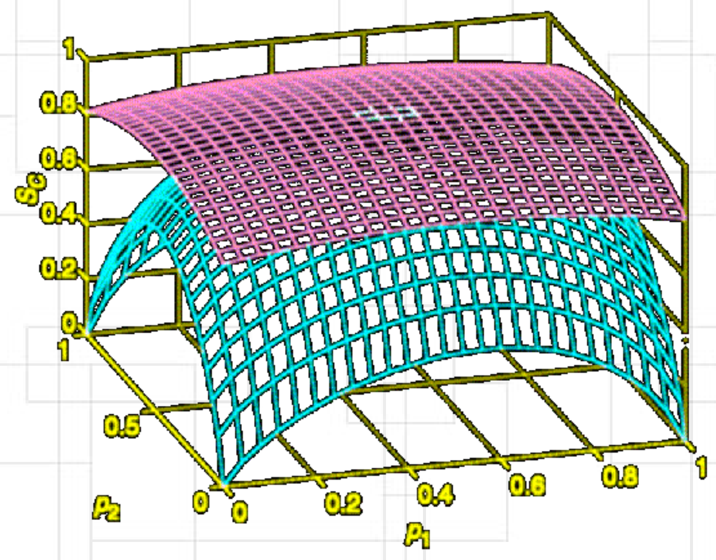


Figure 7. Entropies of the chain generated by game B by considering window of lambda bits. Lambda = 1 is the uncorrelated entropy while lambda = infinity is the correlated entropy.

This was noticed by Shannon for passages of English text. He calculated the entropy just considering every letter, then considering all the pairs, then the triplets and so on. He noticed that the entropy started to asymptotically approach a particular value, the entropy of the English language, which obviously can't be calculated exactly.

Figure 8. The correlated entropies of games B (lower cyan surface) and C (upper magenta surface).



Comparing the entropy rates from games B and C with (considered as sources), when p = 1/2, H_C >= H^B$ as shown in Figure 8. If p ~= 1/2 then for some values of p1 and p2 H_C < H_B. This makes sense if we consider game C to be game B plus another source. If this other source is completely random (p = 1/2), then we are only adding disorder to game B, and cannot decrease the entropy rate.

One way to think of this is as a new paradox in terms of uncorrelated entropy rates: with epsilon = 0, games A and B separately create sequences with maximum uncorrelated entropy rate. However the mixing of A and B creates a sequence with a smaller uncorrelated entropy rate.

This paradox does, however, have a very easy solution: there is no reason to think that mixing games with maximal uncorrelated entropy rate should produce another game with maximal uncorrelated entropy rate in the presence of correlation.

Previous Page

Ratchet and Pawl
Brownian Ratchet
Parrondo's Games
Mathematical Analysis
Parameter Space
Entropy

Next Page

Relevant Papers
Press Articles

Index

A list of papers about Parrondo's games and Brownian ratchets.

1. GP Harmer and D Abbott, ***Parrondo's Paradox***, 1999.
2. GP Harmer, D Abbott, PG Taylor, ***The Paradox of Parrondo's Games***, 2000.
3. GP Harmer, D Abbott, PG Taylor and JMR Parrondo, ***Parrondo's Paradoxical Games and the Discrete Brownian Ratchet***, 2000.
4. GP Harmer, D Abbott, PG Taylor, CEM Pearce and JMR Parrondo, ***Information Entropy and Parrondo's Discrete-Time Ratchet***, 2000.
5. GP Harmer & D Abbott, ***Losing strategies can win by Parrondo's paradox***, 1999.
6. JMR Parrondo, GP Harmer and D Abbott, ***New paradoxical games based on Brownian ratchets***, 2000. Submitted.

Papers written by others that are about or related to Parrondo's games and Brownian ratchets

1. CEM Pearce, ***On Parrondo's Paradoxical Games***, 2000.
2. CEM Pearce, ***Entropy, Markov Information source and Parrondo Games***, 2000.
3. JMR Parrondo and B Jiménez, ***Paradoxical Games and Brownian thermal engines***, Submitted, 2000.
4. Ronald Pyke, ***On Random Walks Related to Parrondo's Games***, In preparation / Submitted, 2000
5. PG Taylor, ***A History-Dependent Parrondo Game***, Submitted, 2000.
6. ES Key, MM Klosek and D Abbott, ***On Parrondo's Paradox: how to construct unfair games by composing fair games***, Submitted, 2000.

The abstracts and reference details for each paper can be viewed at the publications page.

Ratchet and Pawl
Brownian Ratchet
Parrondo's Games
Mathematical Analysis
Parameter Space
Entropy

Next Page

Previous Page

Relevant Papers
Press Articles

Index

Page last updated 13 July, 2000

Page by GPH

Here is a list of news/magazine articles that have appeared regarding the games. They are listed in approximately chronological order.

GACETA COMPLUTENSE -- INVESTIGACIÓN
Un modelo matemático que da mucho juego
La paradoja de Parrondo: perder + perder = ganar (Spanish)
The paradox of Parrondo: to lose + to lose = to win (Bad English translation)

Juan Manuel Rodriguez Parrondo, titular professor of the Department of Atomic, Molecular and Nuclear Physics of the Faculty of Physical Sciences, has devised two simple mathematical games of chance that are interesting to experts in very diverse areas of science. Their results are surprising in statistical terms, to play anyone of the two separately supposes to lose. However, if the player alternates both, in certain or random combinations, it wins. An effect that in articles specialised already is known like "the Parrondo's Paradox". By JAVIER OCHOA.

physics
Good news for losers

If you feel that life always deals you a bad hand, take heart. Some games that you're guaranteed to lose produce surprises if played together, explains PHILIP BALL.

Untitled Piece

Research finds 2 losing games can make a gambler a winner. Look out Las Vegas, here comes Parrondo's paradox. By SUE GOETINCK.

nauka
Skazany Na Sukces?

Polish article, sorry, no interpretation. (29/12/00) By PIOTR CIESLINSKI.

El resultado de juntar dos cosas negativas puede ser positivo (Spanish)

The turn out to join two negative things can be positive (Bad English translation)

There are times in which two bad followed results can give rise one good one: it says the paradox to it of Parrondo. Juan Manuel Rodriguez Parrondo, of 36 years, physicist, professor of the Complutensian University of Madrid, has created two games of chance that more and more intrigue peculiar engineers, mathematicians, biologists and in general. By MÓNICA SALOMONE

Lady Luck: treat her bad and still be glad

Need a bit of luck in the new millennium? A couple of scientists might just have found the key. By SHARON NIXON.

mathematics
Losing to win
online features -- MathTrek
Losing to win

Two games of chance, each guaranteed to give a player a predominance of losses in the long term, can add up to a winning outcome if the player alternates between the two games. By I.P. (15-1-00)

science/health
Paradox in game theory: Losing strategies that win

A Spanish physicist has discovered what appears to be a new law of nature that may help explain, among other things, how life arose out of a primordial soup, why President Clinton´s popularity rose after he was caught in a sex scandal, and why investing in losing stocks can sometimes lead to greater capital gains. By SANDRA BLAKESLEE.

quest/physics
Alternate game play ratchets up winning: It's the law

Similar to above. By SANDRA BLAKESLEE.

Unknown title

Similar to above.(27/1/00, page 8). By SANDRA BLAKESLEE.

Il paradosso di Parrondo, l'affascinante teoria di un fisico spagnolo (Italian)
The paradox of Parrondo, the fascinating theory of a Spanish physicist (Bad English Translation)

From the gamblings we learn to win. (5/2/00). By ROBERTO VACCA.

Wer zweimal verliert, gewinnt (German)
Who loses twice, wins (Good English Translation)

Parrondo's paradox: Confusing for laymen, obvious for mathematicians, but unfortunately useless for players. (7/2/00). By VON JOACHIM LAUKENMANN.

Gambling study show it's good to be bad

Gambling, genetics, the economy and swinging voters - what do they all have in common? By DAVID ELLIS.

cool math site of the week (7-3-00)
knot 199 -- Parrondo's Games

A fine demonstration of the paradoxical Parrondo's game (to lose + to lose = to win!) and the related Brownian Ratchet. Canadian Mathematical Society.

Hasard mathématique et chaos biologique
Qui perd gagne (French)

Mathematical chance and biological chos
Who loses gains (Bad English translation)

Try your chance with a game of chance. Generally, you lose. Play two games of chance, alternatively and in a random way: surprised, you gain! This paradox lights the apparently chaotic mechanisms, and yet oiled well, of the cells or proteins. By HERVÉ RATEL.

Winning With Losing Games:
A New Paradox in the World of Probability

There's an old story about a store owner who loses money on each individual sale but somehow makes it up in volume of sales. By JOHN ALLEN PAULOS.

Update
Gambling on mistakes

Although the odds are stacked against you at both roulette and blackjack there's a sure way to win -- play two games of one, and then shift to the other for two games and then shift back again. By STEPHEN LUNTZ.

Features
Playing both sides

Parrondo's paradox shows that you can win at two losing games by switching between them. The result has surprising implications for the origins of life. By ERICA KLARREICH.

Ratchet and Pawl
Brownian Ratchet
Parrondo's Games
Mathematical Analysis
Parameter Space
Entropy

Next Page

Previous Page

Relevant Papers
Press Articles

Index